Common Trends and Real Exchange Rates

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Abstract The theory of Purchasing Power Parity (PPP) has worked poorly during the post-Bretton Woods period. We generalize the concept of PPP (called Generalized-PPP) and posit an equilibrium relationship among groups of real exchange rates. The basic tenants of G-PPP are that real fundamental macroeconomic shocks tend to be non-stationary so that the real exchange rates themselves will tend to be non-stationary. Although bilateral exchange rates are non-stationary, they will be cointegrated if the vector of stochastically trending variables has reduced rank. G-PPP will hold within the domain of a currency area since the individual nations will experience a set of common real macroeconomic shocks. Using data from the industrialized countries during the post-Bretton Woods period, we show that G-PPP holds for various groupings of nations. We estimate the long-run equilibrium relationships among the real exchange rates and the short-run dynamics concerning the international transmission of real disturbances. An interesting finding is that G-PPP does not hold among the set of major European nations. The direst implication is that such nations do not constitute the domain of a currency area.

1. Introduction

Purchasing Power Parity (PPP) is a convenient analytical simplification since it directly links non-stationary national price levels and exchange rates. The theory is used to transform domestic and foreign nominal magnitudes into similar units and as an element in some theoretical models of exchange rate determination. If p *and* p^* denote the logs domestic and foreign price levels and e denotes the log of the price of foreign exchange, long-run PPP implies that these variables are cointegrated in these sense of Engle and Granger (1987), or, in other words, that the "real" exchange rate $(e + p^* - p)$ is stationary. Numerous studies have shown, however, that real exchange rates are non-stationary calling into question the validity of PPP as a structural model of real exchange rate behavior.[1]

In spite of the questionable evidence for long-run PPP, it is reasonable to expect that the dynamic properties of real exchange rates are linked; the various real exchange rates themselves may be determined by similarly trending stochastic forcing variables. The non-stationarities in a vector process can be modeled in terms of a vector of common stochastic trends (Stock and Watson 1988). The number of stochastic trends, or common long-run components, is equal to the dimension of the vector process less the number of linear independent cointegrating vectors. This implies that a system of nonstationary real exchange rates have a common long-run equilibrium path. In this paper we allow for an equilibrium relationship among groups of real exchange rates, specifically those of the G-7 countries. Cointegration among various real rates is a weaker condition than the stationary of real rates, although PPP results as a special case. The existence of common trends, however, does allow some interesting observations to be made on the nature of modeling the real exchange rate in an interdependent multi-country world.

The rest of the paper is structured as follows. In order to motivate the multivariate cointegration analysis, Section 2 presents univariate tests of bilateral PPP for the G-7 countries. The testing procedure used follows that suggested by Phillips and Perron (1988). Section 3 outlines the common trends approach to modeling a vector of nonstationary time series. The testing procedure for determining the number of common trends, suggested Johansen (1988), is outlined briefly. Section 4 presents evidence of common trends in systems comprising the G-7 group of countries for real exchange rates expressed with the U.S. as the base country.

2. Testing for Pure PPP

Using monthly data from the IMF data tapes, we obtained wholesale prices and exchange rates for the G-7 countries over the period 1973:1 - 1989:12.[2] Bilateral real exchange rates for all country pairs were constructed. Unfortunately, there is no completely adequate method of measuring bilateral real exchange rates. We construct the measure as

$$r_{jkt} = e_{jkt} + p_{kt} - p_{jt} \tag{1}$$

from wholesale prices, p, and nominal exchange rates, r.[3] For example, the real rate between the U.K. and the U.S. is the log of the U.K. price level divided by the product of the U.S. price level and the pound price of the dollar.

The statistics used to test for the stationarity of real exchange rates (and hence for bilateral PPP) are those suggested by Phillips and Perron (1988). The testing strategy is based on the regressions

$$\underline{r}_t = \beta_0 + \rho \underline{r}_{t-1} + \epsilon_t \tag{2}$$

$$\underline{r}_t = \beta_0 + \beta_1 t + \rho \underline{r}_{t-1} + \epsilon_t \tag{3}$$

The standard Dickey-Fuller procedure for testing for unit roots (Dickey and Fuller 1979, 1981) involves testing the hypothesis $\beta_0 = 0$, $\rho = 1$ in equation (2) and $\beta_0 = 0$, $\beta_1 = 0$, $\rho = 1$ in equation (3), on the assumption that the respective error terms are *iid*. Since the disturbances may not be orthogonal or homogenous we use the Phillips and Perron (1988) modifications to the normal Dickey-Fuller test statistics which allow for weakly dependent and heterogeneously distributed innovations.

As a pre-test, we first used the Dickey and Pantula (1987) strategy to test for the possibility of two unit roots. The method involves testing for a unit root in the first-differences of the real exchange rates. At conventional significance levels, we could reject a null of two unit roots for all country pairs. The rejection of the null hypothesis of a unit root in the first-differences implies that the maintained hypothesis becomes a null of a unit root in the real exchange rate series. Rejecting this null would mean that the real exchange rate is stationary.

The impression of non-stationarity is clear on examination of Figure 1. The real exchange rate series shown meander without any tendency to revert to a long-run mean. The formal tests revealed that it is not possible to reject the null of non-stationarity for all real bilateral real exchange rates except that of Italy/Germany.

Table 1: Phillips-Perron Z-values

	Can	Fra	Ger	Italy	Jap	UK
Fra	-1.1					
Ger	-1.8	-2.1				
Italy	-1.9	-2.1	-3.1*			
Jap	-2.0	-1.3	-2.0	-2.5		
UK	-1.5	-0.8	-1.7	-2.3	-2.4	
US	-2.0	-0.9	-1.5	-1.4	-1.7	-1.4

For example, the sample values of the Phillips-Perron $Z_{t,\rho}$ values for $\rho = 1$ shown in Table 1, are all below the 5% critical value except for the Italian/German rate.

When trend stationarity is tested for, it is again the real exchange rates involving Italy which show the strongest evidence of stationarity.[4] Historically, Italy has had differing rates of inflation from the major economies and also from her European partners despite the post-1979 discipline of the EMS exchange rate system. It is also worth remembering that the lira, upon the formation of the EMS was able to float within the wide 6% band allowed by the exchange rate mechanism. The results seem to reinforce Mussa's (1979) assessment that PPP works best for nations experiencing very different rates of inflation.

In cases involving the G-3 countries, U.S., Japan and Germany, PPP is rejected. Furthermore, all the real exchange rates expressed against the U.S. dollar are nonstationary. In the subsequent sections, which use multivariate analysis, we choose the U.S., the largest of the G-7 countries, as the base for expressing the real exchange rate data. The results of this section show that the vector, $\underline{r}_{it}$, which expresses the real exchange rates of country i with respect to the U.S., is known to comprise elements which are all integrated of order *1*, or *I(1)*.

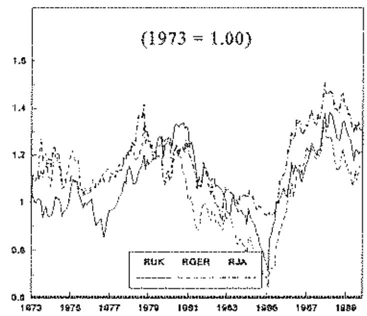

Figure 1: Selected Real Exchange Rates

In general these results must call into question the advisability of using PPP as a theoretical model of exchange rate behavior. This result is not particularly surprising as nonstationary real exchange rates are consistent with many theoretical models of exchange rate behavior. For example, in variants of the Dornbusch (1976) "*overshooting*" model, whilst nominal shocks are neutral in the long-run and hence are consistent with PPP, permanent real shocks can produce permanent movements in real exchange rates. The nonstationarity of real exchange rates is consistent with theoretical exchange rate models of this type to the extent that there have been permanent real shocks.

3. Testing for Common Stochastic Trends

Despite the fact that domestic economies differ, the trends in real forcing variables affecting real exchange rates might be similar across diverse countries. The presence of common trends in both the domestic and international environment might mean that real exchange rates of different countries have a well defined relationship with each other. Deviations from PPP are persistent but there does appear to be some evidence for the hypothesis that the real exchange rates track together. The idea that a non-stationary series may have an equilibrium relationship with other non-stationary series is captured by the concept of cointegration (Engle and Granger 1987). As you can see in Figure 1, there does appear to be a tendency for the real exchange rates to move together even though each is non-stationary.

Cointegration restrictions within a vector of $I(1)$ variables may be tested for in terms of the common trends representation (Stock and Watson 1988). In general the common trends representation may be denoted by

$$\underline{r}_t = \Theta \mu_t + \epsilon_t \qquad E[\epsilon_t] = 0 \;\; E[\eta_t] = 0$$

$$\mu_t = \mu_{t-1} + \eta_t \qquad var[\epsilon_t] = \Sigma_\epsilon \;\; var[\eta_t] = E_\eta \tag{4}$$

where μ_t is a $(k \times 1)$ vector of stochastic trends and Θ is an $(n \times k)$ matrix of parameters. If there are k such stochastic trends $(k < n)$ the model has the important property that n-k linear combinations of $\underline{r}_t$ are stationary. This may be demonstrated by premultiplying (4) by an $((n\text{-}k) \times n)$ matrix B such that

$$B\Theta = 0 \tag{5}$$

If (5) holds then (4) reduces to

$$B\underline{r}_t = B\epsilon_t \tag{6}$$

which is an $((n\text{-}k) \times 1)$ stationary process with mean zero an covariance matrix $B\Sigma_\epsilon B'$. In other words, if each of the n series in $\underline{r}_t$ is integrated of order 1 but can be jointly characterized by $k \leq n$ stochastic trends, then the vector representation of these series has k unit roots and hence n-k distinct stationary linear combinations or cointegrating vectors. The cointegrating vectors are given by the first n-k rows of the matrix B. Clearly (6) is equivalent to the more familiar cointegration representation (cf. Engle and Granger 1987).

Suppose that the n real exchange rates with the U.S. as the base country are cointegrated with a unique cointegrating vector. This implies a long-run equilibrium relationship of the form:

$$\beta_2 r_{2t} + \beta_3 r_{3t} + \beta_4 r4t + ... + \beta_{nt} r_{nt} = 0 \tag{7}$$

It is clear that in the special case where all the β_i for i = 3...n are zero, equation (7) becomes the traditional PPP long-run relation between domestic prices, foreign prices, and the exchange rate.

The relationship between common stochastic trends and cointegration is significant. If the nonstationary forcing variable of a system of real exchange rates can be represented as a vector of common stochastic trends which has a dimension less than that of the original system, then there is at least one linear combination of the real exchange rates which is stationary. Moreover, the disequilibrium error from the long-run relationship is an important component in next period's change in any of the bilateral real exchange rates.

To test for the number of stochastic trends in the vector process Stock and Watson (1988) propose a method based on the left hand side of (6), $B\underline{r}_t$. To overcome the arbitrary normalizations involved in using least squares to estimate the coefficients of the matrix B they propose that the cointegrating vectors be estimated by the factor loadings corresponding to the principal components of $\underline{r}_t$. The first row of B is the linear combination having the smallest variance with the remaining principal components ranked with increasing variance. Although the Stock and Watson (1988) approach is not sensitive to normalization, the testing procedure is *ad hoc* and not based on likelihood theory (Engle and Yoo 1989).

Johansen (1988, 1989) and Johansen and Juselius (1990) provide a unified approach to estimation and testing for cointegration in a likelihood framework. Although the testing procedure is developed in the context of vector autoregressive models the results are clearly interpretable within the common trends framework. Any common trends representation as in (4) may be expressed as an error correcting mechanism as follows:

$$Z(L)(1\text{-}L)\underline{r}_t = - \Gamma(B\underline{r}_t) \tag{8}$$

Z(L) is an $(n \times n)$ matrix of lag polynomials, B is an ((n-k)x n) matrix of cointegrating parameters as before and Γ is an $(n \times (n\text{-}k))$ matrix of weights with which each cointegrating vector enters the n equations.

It is not possible to estimate B and Γ directly using standard estimation methods. Johansen (1988) and Johansen and Juselius (1990) develop a maximum likelihood procedure to estimate these matrices and also likelihood ratio tests to test for the order of cointegration, r. Since the number of stochastic trends in the system, k, is given by the relationship n-k = r, this is equivalent to testing for the number of common stochastic trends.

The Johansen procedure is based on the matrix of vectors of the squared canonical correlations between the residuals of $\underline{r}_t$ and (1-L)$\underline{r}$ regressed on lagged values of (1-L)$\underline{r}_t$. The cointegrating vectors, the rows of the matrix B are those r vectors which constitute significant cointegrating relationships. The value of r

is obtained using the following test statistics suggested by Johansen (1988):

$$\lambda_{trace}(q) = -T\log(1-\hat{\lambda}_i) \tag{9}$$

$$\lambda_{max}(q, q+1) = -T\log(1-\hat{\lambda}_{q+1}) \tag{10}$$

where $\hat{\lambda}_i$ are the ordered eigenvalues of the matrix of canonical correlations. The first statistic tests the null hypothesis that $r \leq q$ against a general alternative. The second statistic tests the null hypothesis $r = q$ against the alternative $r = q + 1$.

4. Cointegrated Real Exchange Rates

We consider first the results of the multivariate cointegration analysis for the G-7 countries with the U.S. as the base. Using the values tabulated in Johansen and Juselius (1990), both the λ_{trace} and the λ_{max} statistics indicate a unique cointegrating vector at the 5% level of significance. The results suggest that the real exchange rates of the G-7 countries are tied together by a unique long-run equilibrium relationship. The long-run relationship before any arbitrary normalization is:

$$-0.72^{*}r_{can} - 0.89^{*}r_{FR} - 1.23^{*}r_{GER} + 3.03^{*}r_{IT} + 1.1956^{*}r_{JAP} - 1.24^{*}r_{UK} -.016739 = 0 \tag{11}$$

The signs of the equilibrium regression indicate that in the long-run, the U.S./Italy and U.S./Japan rates tend to move in opposite directions from the others. In order to test the significance of the terms entering this cointegrating vector all the coefficients were tested for the imposition of a zero restriction. Interestingly enough, despite the clear indication of a unique cointegrating vector for all the G-7 real exchange rates, the coefficient on the Canadian real exchange rate accepted the restriction at the 1% level.

The adjustment coefficients can be interpreted as the speed-of-adjustment toward the long-run equilibrium. The absolute values of the coefficients, particularly for Germany, are small, a result which suggests that adjustment to the long-run equilibrium path is fairly sluggish. Although the signs on the U.K. and French real rates do not have the familiar negative sign for the error correction term, the system as a whole is stable as all the eigenvalues are less than unity in absolute value.

Eliminating Canada and Italy from the system of exchange rates and considering the world's five largest economies yielded disappointing results in terms of the long-run properties of the system. Our formal tests indicate that it is only the λ_{max} statistic which indicates a unique cointegrating vector at the 10% level of significance. Once again the eigenvalues of the system are all less than unity in absolute value indicating system stability, it does not appear that the G-5 countries form a cointegrating set. The failure to find a cointegrating vector is clearly linked to the elimination of Italy from the system.

The situation for the G-3 countries appears more promising. Once again before any normalization the long-run equilibrium relationship is given by

$$-0.96^{*}r_{GER} + 1.77^{*}r_{JAP} - 0.14 = 0 \tag{12}$$

The λ_{trace} statistic indicates acceptance of the above cointegrating vector at the 5% level of significance, whilst the λ_{max} test statistic is marginal at the 5% level. The estimated cointegrating coefficients have the opposite signs and appear to be stable relative to the those of the G-5 nations.

The results for the G-7 and G-3 country groups at least indicate that the standard of construct of bilateral PPP is too simplistic. The implication which follows here is that a nations price level and its nominal bilateral exchange rate with a second country within these groupings are influenced by the real exchange rates of all other countries in the appropriate system.

The empirical work does provide directions for future research on exchange models. At the empirical level it suggests that an autoregressive representations in first differences for any vector of nonstationary real exchange rates is an incorrect specification. When modeling the exchange rates of the G-7 countries as a system it is not appropriate simply to first difference the data as this imposes too many unit roots on the system. At a more theoretical level the results suggest that real exchange rates react not merely to their own particular set of forcing variables but by driving fundamentals which are common to a set of countries.

5. Conclusions

The empirical support for PPP is limited; it works over very long periods or during periods of high inflation. In the sample considered here, real exchange rates are, in general, nonstationary. There is, however, substantial support for the existence of a unique cointegrating vector for a system of $I(1)$ real exchange rates. Using the Johansen (1988) and Johansen and Juselius (1990) testing procedure, we show that there is a unique long-run equilibrium relationship among the real exchange rates of the G-7 countries and the G-3 countries, with the result for the G-5 countries being less clear.

These results have implications for applied and theoretical work in international finance. Most importantly they suggest that a two country model may be too simple to explain the short-run behavior of exchange rates and national price levels. The existence of a long-run equilibrium relationship means that a shock in any one real exchange rate will affect other bilateral exchange rates. If this is the case then most empirical exchange rate models are misspecified; as real exchange rates are linked by a single long-run equilibrium relationship a shock to any one rate will affect the long-run values of others. In other words, the disequilibrium error in terms of the long-run relationship is an important component of the future behavior of the system of real exchange rates. As a consequence, structural models of exchange rate behavior should be enhanced by incorporating the notion of a long-run equilibrium path.

6. References

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7. Footnotes

1. For example, see Adler and Lehman (1983), Corbae and Ouliaris (1988), Enders (1988), Enders and Hurn (1994), and Mussa (1979).
2. The data ends prior to the collapse of the European Monetary System. Similar results to those reported below hold using a longer data span.
3. Wholesale prices are widely available measures and are generally deemed to be more appropriate for international comparisons than consumer price indices. Nominal exchange rates are monthly period averages (line rf of the IMF CD-ROM data) in terms of the US dollar. Cross rates are obtained from the ratio of two US dollar bilateral rates.
4. To conserve space, the tables for the unit root tests including trend and the formal cointegration tests discussed below are not reported here. Details are available from the authors on request.